Is Harmony at the Heart of Things?

Virtually all civilizations, from the Greek and the ancient Mayan to our own, are united by a determined quest for evidence of harmony in the cosmos.

by Anthony Aveni

On January 1, 1801, the first night of a new century, the renowned Sicilian astronomer Giuseppe Piazzi turned his telescope toward a point in the sky between Mars and Jupiter. The faint object he found, exactly where his calculations had predicted, was the first asteroid ever identified. He named it Ceres, after the Roman goddess and protector of his native island. A year later, a German astronomer sighted a second asteroid, which he called Pallas. Its slow but perceptible drift against the background field of distant stars was a dead giveaway that it, too, was a relatively nearby celestial body orbiting the sun. By 1890 astronomers had identified more than 300 asteroids, ranging in size from the giant Ceres, some 500 miles in diameter, to much smaller chunks of rock. Today, with the Hubble space telescope in place, we can track millions of them, all floating in a wide belt between 200 million and 400 million miles from the sun—an unnerving vision at a time when most scientists have come to agree that it was the impact of a single errant asteroid that did in the dinosaurs. What if, we ask ourselves, another asteroid comes hurtling toward Earth?

But the human experience with asteroids so far has much more to tell us about harmony than about apocalypse. One of the more interesting things about asteroids is the unusual way nature has arrayed them in space, and one of the more interesting things about human beings is revealed by our insistent search for an explanation of this arrangement. It is a search strongly rooted in our ancient intuited sense that all things in nature operate rhythmically. Taken to the extreme (which is where I fully intend to carry it), this universal rhythm-seeking reveals nothing less than humanity's age-old attempt to penetrate the mind of God. But let's start with the asteroids.

Several great "scholars of the skies," including Galileo (with pointer) and Copernicus (third from right), share in the search for harmony in the cosmos in a 19th-century print.


ATLAS COELESTIS
seu
HARMONIA
MACROCOSMICA

In the 1850s, when astronomers plotted all the asteroid orbits they'd thus far discovered, they noticed a curious pattern: There were about a dozen gaps in the asteroid belt, "forbidden zones" that the asteroids seemed to shun. Today, we might liken these gaps to the blank bands separating the songs on an old LP record.

What could explain them? The answer didn't come until 1866, when Indiana University astronomer Daniel Kirkwood happened upon a curious coincidence. If there had been asteroids in the gaps, Kirkwood found, there would have been a direct relationship between the time it took each of them to travel around the sun and the time it takes the giant planet Jupiter to do the same. (Not coincidentally, Jupiter is the nearest object large enough to exert a strong gravitational force on the asteroids.) The relationships could be expressed as fractions. Moreover, these fractions were always composed of small whole numbers: one-half, two-thirds, three-fifths, etc.

From there, it was but a few relatively simple steps to understand how Jupiter would pull asteroids in the forbidden zones—which are now called Kirkwood's Gaps—out of their orbit. Imagine that you and I are runners on a circular track and that we start out simultaneously on a half-mile run. Say I complete it in three minutes while you, a faster runner, do it in two (or two-thirds my time). In other words, in the time it takes you to make a full revolution, I can manage only two-thirds of a circuit. If a TV camera in a Goodyear blimp flying overhead follows you from some arbitrary 12 o'clock position on the track all the way around back to that position again, it will show me going only as far as the eight o'clock position. If we continue running at our established paces, once more around the track puts you back to that same 12 o'clock position after four minutes of running but finds me plodding only as far as the 4 o'clock point. At the end of your third revolution, six minutes into the race, you will have gained a full lap, overtaking me at precisely the 12 o'clock point, where I have just completed only my second lap.

Next let's suppose that you are completing the circuit not in some simple fraction of my time but in one made up of larger numbers, such as 11/13. By playing with the hands of a clock, we can see that it will take many more laps before we encounter each other on the same part of the track. (If you're theoretically minded, there is a simple mathematical formula in most elementary astronomy texts you can use to figure this out. The answer turns out to be once every six and one-half of the faster runner's laps.) As a general rule, the smaller the numbers that make up these fractional periods, the more frequent the close encounters.

Now switch back from track stars to real stars, and Kirkwood's Gaps seem less of a mystery. The gaps exist because asteroids that once may have traveled in these vacant zones would have lapped Jupiter more frequently in their

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orbit around the sun, thus bringing them under the giant planet's strong gravitational influence more often. Eventually, gravity prevailed, jerking them out of orbit. (The same logic explains the gaps between the rings of Saturn, with the killer gravitational force supplied by the nearby moon Mimas.) In astronomical parlance, Kirkwood's Gaps are caused by *commensurations* between the periods of asteroids and the period of Jupiter.

"All is number," according to a dictum attributed to Pythagoras (circa 580–500 B.C.).

An interesting word, commensurate. Literally, it means having a common measure, or divisible by a common unit a whole number of times. This combination of parts into a consistent arrangement to form a whole creates what we call *harmony*. Order of this kind pleases the senses, as in the balanced combination of hues that brings joy or satisfaction to the eye by producing harmonious colors. We describe colors that seem to blend in an orderly way as "going together" or "resonating" with one another. In mechanical or electrical systems, resonant vibrations are set up when a periodic stimulus beats in time with the natural frequency of the system. The simplest example I can think of occurs when you push a child on a swing in time with the natural frequency of the swing.

From earliest times, humans have sought harmony and rhythm even where they are not readily perceptible, in fields as varied as astronomy, music, and calendar making. The search for the commensurate, the real subject of this essay, is as old as the oldest religion and far older than the oldest science. It emanates from a time long past, when numbers were thought to have lives of their own.

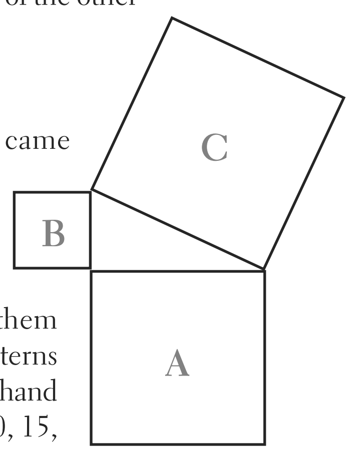
All musicians are aware of the harmonic tones that issue from commensurate lengths of strings we pluck or tubes we blow through. The harmonic principle in music was discovered in the 6th century B.C. by the Greek philosopher Pythagoras. We don't know where he got the idea that number and harmony are linked. One story (probably apocryphal) has it that he heard the sonorous ringing of a blacksmith's hammers of differing weights. But we can be fairly sure that, drawn by curiosity, he eventually took a length of string and marked out on it the proportions 12:8:6. Cutting it into 12:6 and plucking the respective lengths, he heard an octave. The division 12:8 produced a fifth, while the 8:6

resonated in a fourth. These all are consonant chords. (But divide that string 13:11 or 19:12 and you will get a decidedly dissonant chord!) Thus did Pythagoras make the momentous discovery that acoustical consonances are created by commensurate lengths composed of low numbers.*

If all musical sound can be reduced to numbers, Pythagoras wondered, why not other things? The anthropomorphic origin of the number 10 is clear enough—we have 10 fingers and 10 toes. And like the two kinds of numbers, odd and even (or positive and negative), there are two sexes, as well as good and evil. Take the balanced nature of the number four (two times two). Couldn't that represent justice? And why shouldn't six be the number of marriage? (It is the product of 3 and 2, the lowest "male" and "female" numbers.)

Pythagorean inquirers endowed numbers with both a psychological and an ethical dimension. The notion that numbers are the essence of form derives from the Greek love affair with geometry. Though we often think of it as an abstract realm of thought—remember the endless chain of proofs in high school geometry class?—the word *geometry* literally means "land measure." It started out as a practical skill associated with building and farming. Indeed, the celebrated Pythagorean theorem on right triangles is really a formula for finding harmony by equating different areas.

That the square *on* the hypotenuse equals the squares *on* the other two sides of the triangle means that if you make a square, one side of which is the hypotenuse, and two other squares on the remaining two sides of the triangle, the area of the first square is the sum of the area of the other two squares ($C^2=A^2+B^2$), as in the diagram:



The idea that number yields form probably came from the early representation of numbers as dots arranged in patterns. Tallying a large number of items is made simple by visual arrangements. (I remember as a child how quickly I could count up all the pennies in my piggy bank by spreading them out on a large surface, then eyeballing them in patterns of five and sweeping each group with the side of my hand back into the container.) Thus, the numbers 6, 10, 15, 21, 28, etc., are "triangular" because they can be laid out in equilateral triangles. In a bowling alley, for example, the 10 pins are arranged in a 4-3-2-1 pattern. Early numerologists regarded 4, 9, 16, 25, 36, etc., as square, while 6, 12, 20, 30, etc., were thought to be rectangular.

Numbers live! They show their faces in patterns of time as well as space.

*Are such consonant chords artifacts of culture or is the human ear tuned biologically? On this side issue in the age-old nature-nurture debate, the jury is still out. However, some psychologists argue that the tones produced in the simple frequency ratios in a piece by Beethoven or Mozart are naturally more pleasing to the senses than the more complex tones in a modern composition by a Berg or Webern. To prove their point, a group of university scientists recently subjected infants, some as young as four months, to the music of classical and atonal composers. The kids seemed more contented when the harmonious chords of Beethoven's Ninth were played, but they fretted, frowned, and screeched their own dissonant cries as soon as they heard the combined C sharp and F sharp of Schönberg.

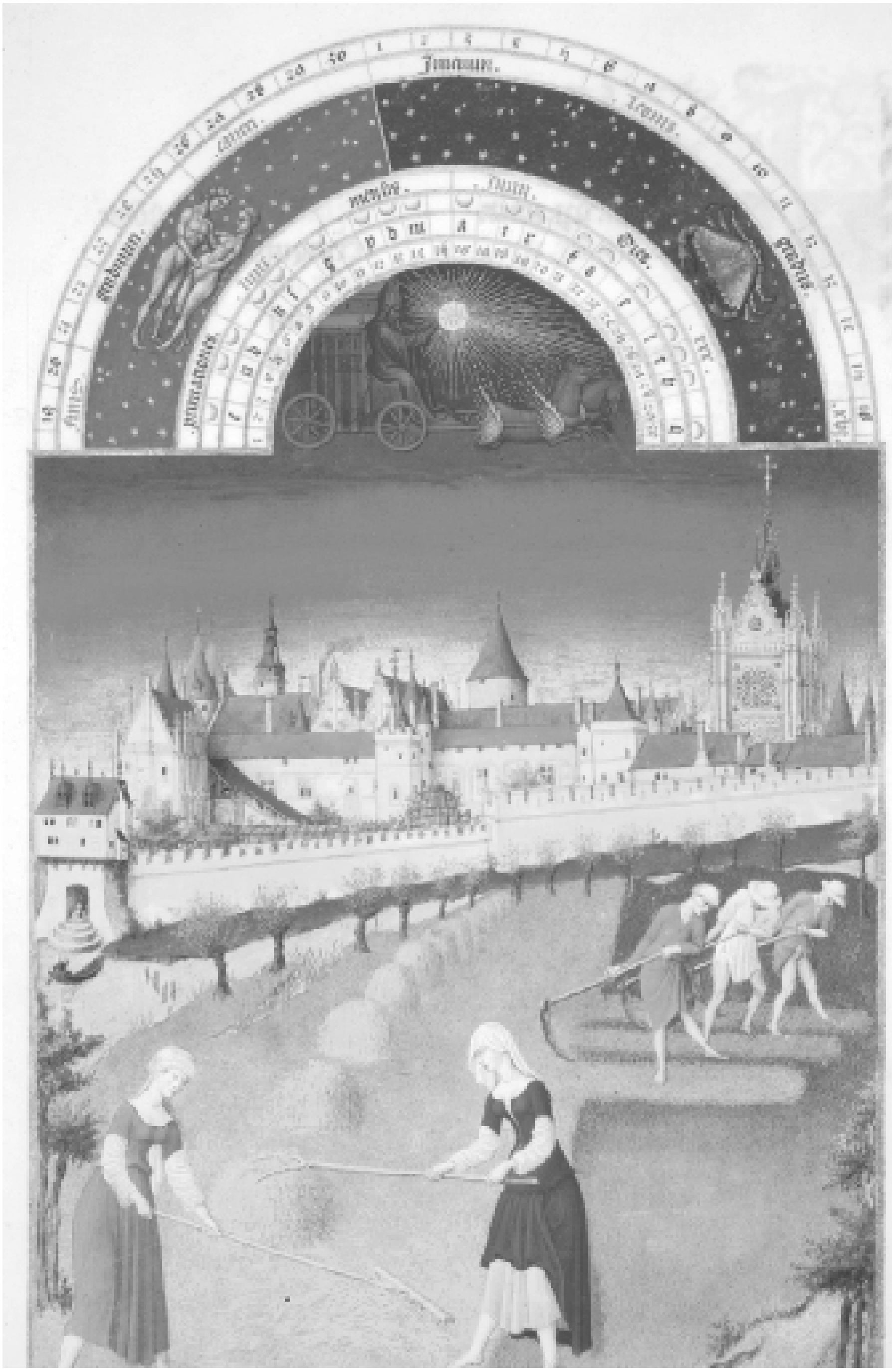
Whether we are dealing with musical harmony or the gravity-produced resonance between missing asteroids and the planet Jupiter, the secret lies in finding the numbers that mesh concordantly, that join together to convince the eye, the ear, or the soul that a degree of order resides in the experience at hand.

Number meets time on the turf of astronomy. One of the basic functions of ancient skywatching the world over lay in the development of calendars. People devised them for various reasons, ranging from the loose demands of agriculture to the more rigid dictates of a state religion. We create calendars to control time. Our desire is to predict the arrival of future events as accurately as possible, literally to reach dates in the future. But to know how nature will behave in the future, we must draw upon the lessons of the past. For clues we can observe the changing position of the Sun at the horizon, the reappearance of the thin crescent Moon, the first morning rise of a bright star or planet, the shortest length of a shadow cast by a stick, or the occurrence of the first rain after a lengthy dry period. But while every calendar begins with a sequence of observed natural events, it is only when these phenomena are related through a numerical correlation that one has a calendar. That's where temporal commensuration begins.

HARMONY PLEASES THE SENSES, AS IN THE BALANCED COMBINATION OF HUES THAT BRINGS JOY OR SATISFACTION TO THE EYE BY PRODUCING HARMONIOUS COLORS.

An early example of this sort of future-date-reaching can be found in the various attempts (I would call them struggles) by the cultures of the world to commensurate the movements of the two primary celestial bodies: the seasonal year of the Sun and the lunar month of the phases. The rising or setting Sun moves through a complete cycle of positions at the horizon in the course of 365.2422 days, while the Moon completes its synodic cycle, from first visible crescent through full and new phase and back again to first crescent, in 29.5306 days. Ancient astronomers reckoned these periods with great precision by repeated observations made over very long intervals.

That these basic time cycles do not naturally mesh is a fact of life. History teaches us that the goal of calendar makers was to invent a harmonic scheme by finding a way to make the cycles fit. How might this work in practice? The solar year is divisible by the synodic month 12 times, with a remainder of 10.8750 days. Suppose we were to begin each month with the occurrence of a first crescent Moon. For simplicity's sake, suppose further that the first of these crescents occurs exactly at the June solstice, when the Sun attains its greatest northerly extreme on the horizon. Recognizing this, calendar keepers would note that the 13th crescent in the lunar cycle would occur some



Elaborate but functional, a 15th-century calendar reminded users that June is the haymaking month.

11 days before the next June solstice, or 354 days later. In other words, in the first solar year, 12 lunar synodic months will have been completed, with a little bit left over. In the second solar year, the 24th crescent in the lunar series would occur about 22 days before the end of the year. By the third solar count, the first crescent would be recorded about 33 days before year's end.

To make things fit better, a calendrical rhythm maker might ask: Why not add a 13th month to the third year to take up the temporal slack? That would result in only three days left over. Following this scheme, the fourth and fifth solar years would consist again of 12 months, but the sixth year would contain 13 months, the last one ending about six days short of the solstice.

This method of inserting extra days or months into the calendar is called *intercalation*. Following the cardinal rule of calendar making—if harmony isn't there, find a way to create it—timekeepers would try to devise a method of intercalation that would guarantee that the lunar and solar years would never get out of step by more than a month. It is easy to see that the simple 12-12-13-12-12-13 method can be further improved by inserting an extra 13-beat measure into the rhythm once the shortfall between first crescent and solstice builds up to a full month. Ancient cultures were thus able to develop some rather impressive intercalation schemes. The leap year schedule in our own calendar is an excellent example of intercalation. It derives from attempts to fit a time period consisting of a whole number of days into a seasonal year made up of a nonwhole number of days.

Such concerns are far distant from the way we think about numbers and time in our daily lives. Ours is a world denuded of the absolute significance of number, thanks in large part to the 17th-century scientific revolution. In one of his dialogues, Galileo (1564–1642) denounced the ancient Greek notion that number, by itself alone, can determine how matter will behave. He put this Pythagorean belief in the mouth of the aptly named Simplicio, who says he believes that the number three is perfect because all complete and whole things in the world have three dimensions as well as three parts (e.g., a beginning, a middle, and an end). Galileo replies through the voice of Salviati—his name is significant too, if you think about it—who scoffs at the notion that a mere number "has a faculty of conferring perfection upon its possessors."

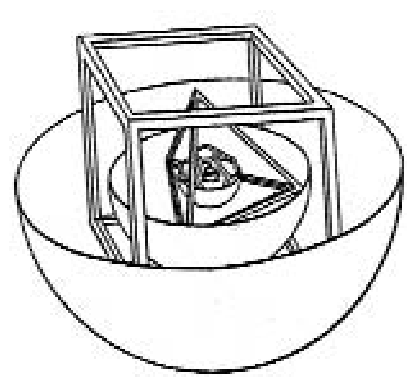
Needless to say, Galileo prevailed. All that remains of the archaic Pythagorean way of thinking about numbers is a lucky 7, an unlucky 13, and "three on a match." The number 10, thoroughly stripped of its divine properties, survives as the base of most of our mathematical systems.

Still, the concept of harmonic numbers found its place in the minds of some early scientists. "There is geometry in the humming of the strings. There is music in the spacing of the spheres." Johannes Kepler, the 17th-century German astronomer, was very much influenced by these words of Pythagoras. He took them to mean that God's secret was encoded in a series of planetary musical tones. Kepler (1571–1630) was convinced that the spheres containing the orbits of the planets are separated by intervals that correspond to the relative length of strings that produce consonant tones, what he called the "*harmonices mundi*" or the "harmony of the spheres."

Kepler dedicated a large portion of his life to studying the positions and motions of the planets, with the goal of determining the sizes and shapes of their orbits. (It was Kepler who discovered that the orbits were elliptical.) Was there a single mathematical or geometrical law, he wondered, that governed a planet's distance from the Sun?

One day, while inscribing a circle inside an equilateral triangle before his class at the University of Graz, in Austria, Kepler is said to have been struck by the idea that the placement of one geometrical figure within another might hold a key to the answer. Kepler knew that there were only five regular polyhedrons (solid figures whose faces are composed of identical polygons): tetrahedrons, cubes, octahedrons, dodecahedrons, and icosahedrons. He was also aware of a famous geometrical proof that demonstrates an essential quality of regular polyhedrons: A sphere can be inscribed within each regular polyhedron such that it touches the center of each face of the polyhedron. Also, spheres can be circumscribed about each of these figures such that the corners of each polyhedron touch the spheres.

Kepler's "eureka moment" came when he realized that there were six planets orbiting the sun (Uranus, Neptune, and Pluto were unknown in the pre-telescopic era) and, consequently, five spaces between them. In his *Astronomia Nova* (1609), he exclaims: "I have brought to life and found true far beyond my hope and expectations that the whole nature of harmonies in the celestial movements really exists—not in the way I thought previously, but in a completely different, yet absolutely perfect manner." Had God deliberately designed the architecture of the universe so that the five regular polyhedrons, each in its correct place, would fit exactly between the planets' orbits around the sun? At the very moment of revelation, according to one version of the story, Kepler dropped his chalk, fled the classroom, and sequestered himself for an intense, lengthy encounter with the axioms of the God-given geometry and numerology of the cosmos. Convinced he was on the right track, Kepler even spent a large portion of his salary to construct a model of the spheres and polyhedrons that fit perfectly one inside the other.



Kepler's model of the solar system

As Kepler later would be forced to admit, his theory about the cosmic significance of the regular polyhedrons was wrong. Never a quitter, the great astronomer tried equating planetary speed with musical pitch. Perhaps the faster planets trilled out high notes while the slower ones growled choral responses in the bass register of the firmament. Together the planets would resonate in a heavenly symphony composed by the Creator. When he attempted to write out God's musical score, Kepler happened upon his harmonic law, the one that correlates a planet's period of revolution with its distance from the Sun. It turned out to be one of the keys to Isaac Newton's discovery of the law of universal gravitation in 1687.

Contemporary historians of science call Newton a genius, while Kepler is often denigrated for having followed the lead of a nonsensical revelation about commensurate geometry. But in the Europe of Kepler's era, it would not have been unreasonable to think of God as a universal craftsman, the divine musical composer who set the planets in motion, each with its own pitch that contributed to the Harmony of the Worlds. And Kepler's quest for the commensurate still resonates. In 1930, on the 300th anniversary of his death, as scientists were exploring the spacing of electrons about atomic nuclei, the physicist Arnold Sommerfeld asked,

> Would Kepler, the Mystic who, like Pythagoras and Plato, tried to find and to enjoy the harmonies of the Cosmos, would he have been surprised that atomic physics had re-discovered the very same harmonies in the building-stones of matter, and this in even purer form? For the integral numbers in the original quantum-theory display a greater harmonic consonance than even the stars in the Pythagorean music of the spheres.

The search for things commensurate—for balance, equilibrium, and harmony that please the senses—hasn't been only a Western pursuit. It lies at the foundation of mathematical systems in cultures all over the world. A case in point is the divine coalescence of numbers derived by the ancient Maya, a culture just about as far removed from our Greek ancestors as we can imagine.

Numeration had great potency in ancient Mayan thought. During the first millennium A.D., Mayan artisans chiseled numbers in stone and painted them in manuscripts, on pots, and on wall-sized murals all over Central America. Among the relics of Mayan civilization are tall, rectangular stones called *stelae*, engraved with highly stylized numbers. Epigraphers think people once stood in front of these monuments chanting the names of their number gods, hoping to influence divine intervention in their lives. Each number was conceived as a god with particular characteristics related to age, sex, sexual prowess, and other aspects of human existence. Thick lipped, his face spotted with tattoos, the god who depicted the number two symbolized death and sacrifice; the wrinkled countenance of number five reminds us of the wisdom of old age. In Mayan society these sacred numbers apparently made the passage of time possible, for the number gods are often shown carrying the burden of the days, parceled out into units (like our days, months, and years), upon their backs.

To comprehend the Mayan numerological mentality, we must listen to the sky. Like the ancient Greeks, we pick up the beat of the two loudest instruments in the firmament, the Sun and the Moon, and then, if we are Maya, listen for the next most audible. It comes from the planet Venus, the third brightest object in the sky. The search for harmony compels us to seek another beat, to create a musical score to which all three luminaries can dance.

What made Venus so special for the Maya was the fact that its cycle of 584 days happens to resonate with the cycle of the seasons, or 365 days, in the perfect ratio of two small whole numbers: eight to five. In practical terms, this means that to the careful eye any visible aspect of Venus timed relative to the position of the sun will be repeated almost exactly eight years later. For example, if Venus first appears as a morning star on the first day of winter 2001, it will repeat that performance very close to that date in 2009 and again in 2017. How satisfying it must have been for the Mayan keeper of the days to find such pristine order in an otherwise chaotic world!

A seasonal index like this one could be useful to any practical-minded people who kept time by a solar-based calendar, especially if they had latched on to expressing periods in whole-number ratios. For a time, the Maya became obsessed with this Venus cycle, recognizing that it also conformed with the moon's phases. The Venus eight-year cycle also equals a whole number of lunar synodic months (99 of them to be exact). So, the phase of the moon that accompanies the first appearance of Venus—say, at the December solstice in 2001—will be repeated around the time of the December solstice eight years later, thus signaling the return of Venus.

The Mayan number god zero carries the burden of days. The figure appears on a stone* stela *found in Honduras.

The euphonious coming together of natural cyclic periods may seem unimportant to us. It scarcely matters, for example, what day of the week coincides with New Year's Day from year to year. But for societies whose systems of timekeeping were based on repetitive natural phenomena, some of them projected all the way back to the mythic creation of the world, the revelation of commensurate quantities underpinning the wanderings of their celestial deities would have been regarded as a major discovery revealing the secrets of the universe.

Mayan philosophers of time were not content only to compose a celestial symphony. They sought rhythm-making numbers linked to other periodicities involving the pulse of their lives, cosmic beats that penetrated their very bodies. For example, they recognized that the length of time Venus spends as a morning or an evening star was approximately equal to the sacred count of 260 days. That number appeared very early (ca. 600 B.C.) in the development of the Mayan calendar, when Mayan timekeepers recognized the near equivalence of the time of human gestation in days and the product of the number of layers in heaven (13) and the number of fingers and toes on the human body (20)—yet another kind of commensuration.

Captivated by the rhythms of life and nature, Mayan seekers of the commensurate apparently would go to any lengths to acquire the magical beat. Let me close by citing a recent discovery in ancient Mayan epigraphy that I believe is as important to the study of the Maya as the discovery of Kirkwood's Gaps was to the rise of 19th-century astronomy. In a sense, the two discoveries resonate with each other.

In the Mayan world, common birth dates implied common attributes, for the date of one's birth was said to ordain one's destiny. To provide a numerological charter attesting to the legitimacy of their rulers, the ancient Maya went to a great deal of trouble to demonstrate that leaders were born on days with the same name as the gods who created the world. So we ought not be surprised to find in the Mayan inscriptions certain large numbers that are exactly divisible by a wide range of natural time cycles. There is on page 24 of the Venus Table in the Dresden Codex (a Mayan hieroglyphic book of divination dated to shortly before the Spanish conquest), a very seminal large number that occupies the starting point in a time-reckoning scheme that accurately tracks the position of the planet Venus in the sky. The magic number is written as an interval said to have elapsed since events that took place in heaven before the creation of the world as we now know it. This number translates from the Mayan base-20 system of counting into 1,366,560 of our days (about 3,741 years). My colleague, the late Yale University linguist Floyd Lounsbury, dubbed it the "super number" of the Mayan codices. I think he had good reason for doing so, because he had discovered, to his amazement, that it is an exact whole multiple of several other numbers of vital interest to the Maya: the period of Venus (584 days), the length of the entire Venus Table (37,960 days), the period of Mars (780 days), the seasonal year (365 days), and the period of Mercury (117 days). And, as might be expected, it is also commensurate with the most sacred of all Mayan cycles, the 260-day count.

I cannot even begin to hazard a guess about how the Maya might have happened upon this "mother of all numbers." It must have taken generations of careful skywatching and years of mathematical calculation to root out the commensurate cosmic number par excellence, the "gravitational constant" in the Mayan universe of numbers. Like the lost chord, such a grand cycle resonating perfect harmony defies all credibility even as it inspires awe.

I have a feeling that all cultures at one time or another taste the passion for perfection derived from questing after the commensurate. I wonder what the Mayan Kepler, enraptured by that eureka moment of discovery, must have thought when the divine cosmic beat suddenly popped out at him. Ptolemy of Alexandria, greatest of all the Greek astronomers, captured the feeling perfectly when, after his own harmonic revelation more than two millennia ago, he wrote that "in studying the convoluted orbits of the stars my feet do not touch the earth, and, seated at the table of Zeus himself, I am nurtured with celestial ambrosia." ❑